

Isaac Newton, *The Principia: Mathematical Principles of Natural Philosophy*, translated by I.B. Cohen and A. Whitman, University of California Press, 1999, pp. 408-415, 791-796, 802-805.

Scholium Thus far it has seemed best to explain the senses in which less familiar words are to be taken in this treatise. Although time, space, place, and motion are very familiar to everyone, it must be noted that these quantities are popularly conceived solely with reference to the objects of sense perception. And this is the source of certain preconceptions; to eliminate them it is useful to distinguish these quantities into absolute and relative, true and apparent, mathematical and common.

1. Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration. Relative, apparent, and common time is any sensible and external measure "(precise or imprecise)" of duration by means of motion; such a measure—for example, an hour, a day, a month, a year—is commonly used instead of true time.

2. Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is an

aa. Newton uses the phrase "seu accurata seu inaequabilis"—literally, "exact or nonuniform."

movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies and is popularly used for immovable space, as in the case of space under the earth or in the air or in the heavens, where the dimension is determined from the situation of the space with respect to the earth. Absolute and relative space are the same in species and in magnitude, but they do not always remain the same numerically. For example, if the earth moves, the space of our air, which in a relative sense and with respect to the earth always remains the same, will now be one part of the absolute space into which the air passes, now another part of it, and thus will be changing continually in an absolute sense.

3. Place is the part of space that a body occupies, and it is, depending on the space, either absolute or relative. I say the part of space, not the position of the body or its outer surface. For the places of equal solids are always equal, while their surfaces are for the most part unequal because of the dissimilarity of shapes; and positions, properly speaking, do not have quantity and are not so much places as attributes of places. The motion of a whole is the same as the sum of the motions of the parts; that is, the change in position of a whole from its place is the same as the sum of the changes in position of its parts from their places, and thus the place of a whole is the same as the sum of the places of the parts and therefore is internal and in the whole body.

4. Absolute motion is the change of position of a body from one absolute place to another; relative motion is change of position from one relative place to another. Thus, in a ship under sail, the relative place of a body is that region of the ship in which the body happens to be or that part of the whole interior of the ship which the body fills and which accordingly moves along with the ship, and relative rest is the continuance of the body in that same region of the ship or same part of its interior. But true rest is the continuance of a body in the same part of that unmoving space in which the ship itself, along with its interior and all its contents, is moving. Therefore, if the earth is truly at rest, a body that is relatively at rest on a ship will move truly and absolutely with the velocity with which the ship is moving on the earth. But if the earth is also moving, the true and absolute motion of the body will arise partly from the true motion of the earth in unmoving space and partly from the relative motion of the ship on the earth. Further, if the body is also moving relatively on the ship, its true motion will arise partly from

the true motion of the earth in unmoving space and partly from the relative motions both of the ship on the earth and of the body on the ship, and from these relative motions the relative motion of the body on the earth will arise. For example, if that part of the earth where the ship happens to be is truly moving eastward with a velocity of 10,010 units, and the ship is being borne westward by sails and wind with a velocity of 10 units, and a sailor is walking on the ship toward the east with a velocity of 1 unit, then the sailor will be moving truly and absolutely in unmoving space toward the east with a velocity of 10,001 units and relatively on the earth toward the west with a velocity of 9 units.

In astronomy, absolute time is distinguished from relative time by the equation of common time. For natural days, which are commonly considered equal for the purpose of measuring time, are actually unequal. Astronomers correct this inequality in order to measure celestial motions on the basis of a truer time. It is possible that there is no uniform motion by which time may have an exact measure. All motions can be accelerated and retarded, but the flow of absolute time cannot be changed. The duration or perseverance of the existence of things is the same, whether their motions are rapid or slow or null; accordingly, duration is rightly distinguished from its sensible measures and is gathered from them by means of an astronomical equation. Moreover, the need for using this equation in determining when phenomena occur is proved by experience with a pendulum clock and also by eclipses of the satellites of Jupiter.

Just as the order of the parts of time is unchangeable, so, too, is the order of the parts of space. Let the parts of space move from their places, and they will move (so to speak) from themselves. For times and spaces are as it were, the places of themselves and of all things. All things are placed in time with reference to order of succession and in space with reference to order of position. It is of the essence of spaces to be places, and for primary places to move is absurd. They are therefore absolute places, and it is only changes of position from these places that are absolute motions.

But since these parts of space cannot be seen and cannot be distinguished from one another by our senses, we use sensible measures in their stead. For we define all places on the basis of the positions and distances of things from some body that we regard as immovable, and then we reckon all motions with respect to these places, insofar as we conceive of bodies as being changed.

in position with respect to them. Thus, instead of absolute places and motions we use relative ones, which is not inappropriate in ordinary human affairs, although in philosophy abstraction from the senses is required. For it is possible that there is no body truly at rest to which places and motions may be referred.

Moreover, absolute and relative rest and motion are distinguished from each other by their properties, causes, and effects. It is a property of rest that bodies truly at rest are at rest in relation to one another. And therefore, since it is possible that some body in the regions of the fixed stars or far beyond is absolutely at rest, and yet it cannot be known from the position of bodies in relation to one another in our regions whether or not any of these maintains a given position with relation to that distant body, true rest cannot be defined on the basis of the position of bodies in relation to one another.

It is a property of motion that parts which keep given positions in relation to wholes participate in the motions of such wholes. For all the parts of bodies revolving in orbit endeavor to recede from the axis of motion, and the impetus of bodies moving forward arises from the joint impetus of the individual parts. Therefore, when bodies containing others move, whatever is relatively at rest within them also moves. And thus true and absolute motion cannot be determined by means of change of position from the vicinity of bodies that are regarded as being at rest. For the exterior bodies ought to be regarded not only as being at rest but also as being truly at rest. Otherwise all contained bodies, besides being subject to change of position from the vicinity of the containing bodies, will participate in the true motions of the containing bodies and, if there is no such change of position, will not be truly at rest but only be regarded as being at rest. For containing bodies are to those inside them as the outer part of the whole to the inner part or as the shell to the kernel. And when the shell moves, the kernel also, without being changed in position from the vicinity of the shell, moves as a part of the whole.

A property akin to the preceding one is that when a place moves, whatever is placed in it moves along with it, and therefore a body moving away from a place that moves participates also in the motion of its place. Therefore, all motions away from places that move are only parts of whole and absolute motions, and every whole motion is compounded of the motion of a body away from its initial place, and the motion of this place away from

its place, and so on, until an unmoving place is reached, as in the above mentioned example of the sailor. Thus, whole and absolute motions can be determined only by means of unmoving places, and therefore in what has preceded I have referred such motions to unmoving places and relative motions to movable places. Moreover, the only places that are unmoving are those that all keep given positions in relation to one another from infinity to infinity and therefore always remain immovable and constitute the space that I call immovable.

The causes which distinguish true motions from relative motions are the forces impressed upon bodies to generate motion. True motion is neither generated nor changed except by forces impressed upon the moving body itself, but relative motion can be generated and changed without the impression of forces upon this body. For the impression of forces solely on other bodies with which a given body has a relation is enough, when the other bodies yield, to produce a change in that relation which constitutes the relative rest or motion of this body. Again, true motion is always changed by forces impressed upon a moving body, but relative motion is not necessarily changed by such forces. For if the same forces are impressed upon a moving body and also upon other bodies with which it has a relation, in such a way that the relative position is maintained, the relation that constitutes the relative motion will also be maintained. Therefore, every relative motion can be changed while the true motion is preserved, and can be preserved while the true one is changed, and thus true motion certainly does not consist in relations of this sort.

The effects distinguishing absolute motion from relative motion are the forces of receding from the axis of circular motion. For in purely relative circular motion these forces are null, while in true and absolute circular motion they are larger or smaller in proportion to the quantity of motion. If a bucket is hanging from a very long cord and is continually turned around until the cord becomes twisted tight, and if the bucket is thereupon filled with water and is at rest along with the water and then, by some sudden force, is made to turn around in the opposite direction and, as the cord unwinds, perseveres for a while in this motion; then the surface of the water will at first be level, just as it was before the vessel began to move. But after the vessel, by the force gradually impressed upon the water, has caused the water also to begin revolving perceptibly, the water will gradually recede

from the middle and rise up the sides of the vessel, assuming a concave shape (as experience has shown me), and, with an ever faster motion, will rise further and further until, when it completes its revolutions in the same times as the vessel, it is relatively at rest in the vessel. The rise of the water reveals its endeavor to recede from the axis of motion, and from such an endeavor one can find out and measure the true and absolute circular motion of the water, which here is the direct opposite of its relative motion. In the beginning, when the relative motion of the water in the vessel was greatest, that motion was not giving rise to any endeavor to recede from the axis; the water did not seek the circumference by rising up the sides of the vessel but remained level, and therefore its true circular motion had not yet begun. But afterward, when the relative motion of the water decreased, its rise up the sides of the vessel revealed its endeavor to recede from the axis, and this endeavor showed the true circular motion of the water to be continually increasing and finally becoming greatest when the water was relatively at rest in the vessel. Therefore, that endeavor does not depend on the change of position of the water with respect to surrounding bodies, and thus true circular motion cannot be determined by means of such changes of position. The truly circular motion of each revolving body is unique, corresponding to a unique endeavor as its proper and sufficient effect, while relative motions are innumerable in accordance with their varied relations to external bodies and, like relations, are completely lacking in true effects except insofar as they participate in that true and unique motion. Thus, even in the system of those who hold that our heavens revolve below the heavens of the fixed stars and carry the planets around with them, the individual parts of the heavens, and the planets that are relatively at rest in the heavens to which they belong, are truly in motion. For they change their positions relative to one another (which is not the case with things that are truly at rest), and as they are carried around together with the heavens, they participate in the motions of the heavens and, being parts of revolving wholes, endeavor to recede from the axes of those wholes.

Relative quantities, therefore, are not the actual quantities whose names they bear but are those sensible measures of them (whether true or erroneous) that are commonly used instead of the quantities being measured. But if the meanings of words are to be defined by usage, then it is these sensible measures which should properly be understood by the terms "time,"

"space," "place," and "motion," and the manner of expression will be out of the ordinary and purely mathematical if the quantities being measured are understood here. Accordingly those who there interpret these words as referring to the quantities being measured do violence to the Scriptures. And they no less corrupt mathematics and philosophy who confuse true quantities with their relations and common measures.

It is certainly very difficult to find out the true motions of individual bodies and actually to differentiate them from apparent motions, because the parts of that immovable space in which the bodies truly move make no impression on the senses. Nevertheless, the case is not utterly hopeless. For it is possible to draw evidence partly from apparent motions, which are the differences between the true motions, and partly from the forces that are the causes and effects of the true motions. For example, if two balls, at a given distance from each other with a cord connecting them, were revolving about a common center of gravity, the endeavor of the balls to recede from the axis of motion could be known from the tension of the cord, and thus the quantity of circular motion could be computed. Then, if any equal forces were simultaneously impressed upon the alternate faces of the balls to increase or decrease their circular motion, the increase or decrease of the motion could be known from the increased or decreased tension of the cord, and thus finally, it could be discovered which faces of the balls the forces would have to be impressed upon for a maximum increase in the motion, that is, which were the posterior faces, or the ones that are in the rear in a circular motion. Further, once the faces that follow and the opposite faces that precede were known, the direction of the motion would be known. In this way both the quantity and the direction of this circular motion could be found in any immense vacuum, where nothing external and sensible existed with which the balls could be compared. Now if some distant bodies were set in that space and maintained given positions with respect to one another, as the fixed stars do in the regions of the heavens, it could not, of course, be known from the relative change of position of the balls among the bodies whether the motion was to be attributed to the bodies or to the balls. But if the cord was examined and its tension was discovered to be the very one which the motion of the balls required, it would be valid to conclude that the motion belonged to the balls and that the bodies were at rest, and then, finally, from the change of position of the balls among the bodies, to determine

the direction of this motion. But in what follows, a fuller explanation will be given of how to determine true motions from their causes, effects, and apparent differences, and, conversely, of how to determine from motions, whether true or apparent, their causes and effects. For this was the purpose for which I composed the following treatise.

BOOK 3

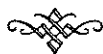
THE SYSTEM OF THE WORLD



In the preceding books I have presented principles of philosophy^a that are not, however, philosophical but strictly mathematical—that is, those on which the study of philosophy can be based. These principles are the laws and conditions of motions and of forces, which especially relate to philosophy. But in order to prevent these principles from seeming sterile, I have illustrated them with some philosophical scholiums [i.e., scholiums dealing with natural philosophy], treating topics that are general and that seem to be the most fundamental for philosophy, such as the density and resistance of bodies, spaces void of bodies, and the motion of light and sounds. It still remains for us to exhibit the system of the world from these same principles. On this subject I composed an earlier version of book 3 in popular form, so that it might be more widely read. But those who have not sufficiently grasped the principles set down here will certainly not perceive the force of the conclusions, nor will they lay aside the preconceptions to which they have become accustomed over many years; and therefore, to avoid lengthy disputations, I have translated the substance of the earlier version into propositions in a mathematical style, so that they may be read only by those who have first mastered the principles. But since in books 1 and 2 a great number of propositions occur which might be too time-consuming even for readers who are proficient in mathematics, I am unwilling to advise anyone to study every one of these propositions. It will be sufficient to read with care the Definitions, the Laws of Motion, and the first three sections of book 1, and then turn to this book 3 on the system of the world, consulting at will the other propositions of books 1 and 2 which are referred to here.

a. In this introduction to book 3, Newton uses "philosophy" and its adjective "philosophical" to refer to "natural philosophy." According to John Harris's *Lexicon Technicum* (London, 1704), natural philosophy is that "Science which contemplates the Powers of Nature, the Properties of Natural Bodies, and their mutual Action one upon another." The half title of the third edition of the *Principia* reads "Newtoni Principia Philosophiae" ("Newton's Principles of Philosophy"). The dedication page of the *Principia*, in all editions, refers to the Royal Society as founded "ad philosophiam promovendam" ("for the promotion of philosophy").

^aRULES FOR THE STUDY OF NATURAL PHILOSOPHY



Rule 1 *No more causes of natural things should be admitted than are both true and sufficient to explain their phenomena.*

As the philosophers say: Nature does nothing in vain, and more causes are in vain when fewer suffice. For nature is simple and does not indulge in the luxury of superfluous causes.

aa. Ed. 1 has nine numbered "Hypotheses," most of which ed. 2 converts into two categories, now called "Rules for Natural Philosophy" and "Phenomena." Hyps. 1 and 2 become rules 1 and 2; hyp. 3 is discarded, to be replaced by rule 3; hyp. 4 becomes hyp. 1 and is transferred to a location between prop. 10 and prop. 11; hyps. 5-9 become phen. 1, 3-6, while phen. 2 is new in ed. 2. Ed. 3 further introduces rule 4. These changes may be tabulated as follows:

<i>Ed. 1</i>	<i>Ed. 2</i>	<i>Ed. 3</i>
hypothesis 1	rule 1	rule 1
hypothesis 2	rule 2	rule 2
hypothesis 3	—	—
—	rule 3	rule 3
—	—	rule 4
hypothesis 4	hypothesis 1*	hypothesis 1*
hypothesis 5	phenomenon 1	phenomenon 1
—	phenomenon 2	phenomenon 2
hypothesis 6	phenomenon 3	phenomenon 3
hypothesis 7	phenomenon 4	phenomenon 4
hypothesis 8	phenomenon 5	phenomenon 5
hypothesis 9	phenomenon 6	phenomenon 6

*between prop. 10 and prop. 11

Ed. 2 also has additions of explanatory phrases and sentences, alterations in wording, and, for the phenomena, revisions of numerical data and references to observers. Ed. 3 further expands or adds some explanatory sentences. For details see the Guide to the present translation, §8.2. Cf. also Alexandre Koyré, "Newton's 'Regulae Philosophandi,'" in his *Newtonian Studies* (Cambridge, Mass.: Harvard University Press, 1965), pp. 261-272; I. Bernard Cohen, "Hypotheses in Newton's Philosophy," *Physis: Rivista inter-*

Therefore, the causes assigned to natural effects of the same kind must be, so far as possible, the same. Rule 2

Examples are the cause of respiration in man and beast, or of the falling of stones in Europe and America, or of the light of a kitchen fire and the sun, or of the reflection of light on our earth and the planets.

Those qualities of bodies that cannot be intended and remitted [i.e., qualities that cannot be increased and diminished] and that belong to all bodies on which experiments can be made should be taken as qualities of all bodies universally. ^bRule 3

For the qualities of bodies can be known only through experiments; and therefore qualities that square with experiments universally are to be regarded as universal qualities; and qualities that cannot be diminished cannot be taken away from bodies. Certainly idle fancies ought not to be fabricated recklessly against the evidence of experiments, nor should we depart from the analogy of nature, since nature is always simple and ever consonant with itself. The extension of bodies is known to us only through our senses, and yet there are bodies beyond the range of these senses; but because extension is found in all sensible bodies, it is ascribed to all bodies universally. We know by experience that some bodies are hard. Moreover, because the hardness of the whole arises from the hardness of its parts, we justly infer from this not only the hardness of the undivided particles of bodies that are accessible to our senses, but also of all other bodies. That all bodies are impenetrable we gather not by reason but by our senses. We find those bodies that we handle to be impenetrable, and hence we conclude that impenetrability is a property of all bodies universally. That all bodies are movable and persevere in motion or in rest by means of certain forces (which we call forces of inertia) we infer from finding these properties in the bodies that we have seen. The extension, hardness, impenetrability, mobility, and force of inertia of the whole arise from the extension, hardness, impenetrability, mobility, and force of inertia of each of the parts; and thus we conclude that every one of the least parts

nazionale di storia della scienza 8 (1966): 163–184, reprinted in *Proceedings of the Boston Colloquium for the Philosophy of Science 1966/1968*, ed. Robert S. Cohen and Marx W. Wartofsky, Boston Studies in the Philosophy of Science, vol. 5 (Dordrecht: D. Reidel Publishing Co., 1969), pp. 304–326; I. Bernard Cohen, *Introduction to Newton's "Principia"* (Cambridge, Mass.: Harvard University Press; Cambridge: Cambridge University Press, 1971), pp. 23–26, 240–245.

bb. Ed. 1 has: "Hypothesis 3. Every body can be transformed into a body of any other kind and successively take on all the intermediate degrees of qualities." Cf. prop. 6, corol. 2, below.

of all bodies is extended, hard, impenetrable, movable, and endowed with a force of inertia. And this is the foundation of all natural philosophy. Further, from phenomena we know that the divided, contiguous parts of bodies can be separated from one another, and from mathematics it is certain that the undivided parts can be distinguished into smaller parts by our reason. But it is uncertain whether those parts which have been distinguished in this way and not yet divided can actually be divided and separated from one another by the forces of nature. But if it were established by even a single experiment that in the breaking of a hard and solid body, any undivided particle underwent division, we should conclude by the force of this third rule not only that divided parts are separable but also that undivided parts can be divided indefinitely.

Finally, if it is universally established by experiments and astronomical observations that all bodies on or near the earth gravitate [*lit.* are heavy] toward the earth, and do so in proportion to the quantity of matter in each body, and that the moon gravitates [*is heavy*] toward the earth in proportion to the quantity of its matter, and that our sea in turn gravitates [*is heavy*] toward the moon, and that all planets gravitate [*are heavy*] toward one another, and that there is a similar gravity [*heaviness*] of comets toward the sun, it will have to be concluded by this third rule that all bodies gravitate toward one another. Indeed, the argument from phenomena will be even stronger for universal gravity than for the impenetrability of bodies, for which, of course, we have not a single experiment, and not even an observation, in the case of the heavenly bodies. Yet I am by no means affirming that gravity is essential to bodies. By inherent force I mean only the force of inertia. This is immutable. Gravity is diminished as bodies recede from the earth.^b

Rule 4 *In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions.*

This rule should be followed so that arguments based on induction may not be nullified by hypotheses.

PROPOSITIONS



- Proposition 1** *The forces by which the circumjovial planets [or satellites of Jupiter] are continually drawn away from rectilinear motions and are maintained in their respective orbits are directed to the center of Jupiter and are inversely as the squares of the distances of their places from that center.*

The first part of the proposition is evident from phen. 1 and from prop. 2 or prop. 3 of book 1, and the second part from phen. 1 and from corol. 6 to prop. 4 of book 1.

The same is to be understood for the planets that are Saturn's companions [or satellites] by phen. 2.

- Proposition 2** *The forces by which the primary planets are continually drawn away from rectilinear motions and are maintained in their respective orbits are directed to the sun and are inversely as the squares of their distances from its center.*

The first part of the proposition is evident from phen. 5 and from prop. 2 of book 1, and the latter part from phen. 4 and from prop. 4 of the same book. But this second part of the proposition is proved with the greatest exactness from the fact that the aphelia are at rest. For the slightest departure from the ratio of the square would (by book 1, prop. 45, corol. 1) necessarily result in a noticeable motion of the apsides in a single revolution and an immense such motion in many revolutions.

- Proposition 3** *The force by which the moon is maintained in its orbit is directed toward the earth and is inversely as the square of the distance of its places from the center of the earth.*

The first part of this statement is evident from phen. 6 and from prop. 2 or prop. 3 of book 1, and the second part from the very slow motion of the moon's apogee. For that motion, which in each revolution is only three

degrees and three minutes forward [or in consequentia, i.e., in an easterly direction] can be ignored. For it is evident (by book 1, prop. 45, corol. 1) that if the distance of the moon from the center of the earth is to the semidiameter of the earth as D to 1, then the force from which such a motion may arise is inversely as $D^{2\frac{1}{243}}$, that is, inversely as that power of D of which the index is $2\frac{1}{243}$; that is, the proportion of the force to the distance is inversely as a little greater than the second power of the distance, but is $59\frac{3}{4}$ times closer to the square than to the cube. Now this motion of the apogee arises from the action of the sun (as will be pointed out below) and accordingly is to be ignored here. The action of the sun, insofar as it draws the moon away from the earth, is very nearly as the distance of the moon from the earth, and so (from what is said in book 1, prop. 45, corol. 2) is to the centripetal force of the moon as roughly 2 to 357.45, or 1 to $178\frac{29}{40}$. And if so small a force of the sun is ignored, the remaining force by which the moon is maintained in its orbit will be inversely as D^2 . And this will be even more fully established by comparing this force with the force of gravity as is done in prop. 4 below.

COROLLARY. If the mean centripetal force by which the moon is maintained in its orbit is increased first in the ratio of $177\frac{29}{40}$ to $178\frac{29}{40}$, then also in the squared ratio of the semidiameter of the earth to the mean distance of the center of the moon from the center of the earth, the result will be the lunar centripetal force at the surface of the earth, supposing that that force, in descending to the surface of the earth, is continually increased in the ratio of the inverse square of the height.

The moon gravitates toward the earth and by the force of gravity is always drawn back from rectilinear motion and kept in its orbit.

**Proposition 4
Theorem 4**

The mean distance of the moon from the earth in the syzygies is, according to Ptolemy and most astronomers, 59 terrestrial semidiameters, 60 according to Vendelin and Huygens, $60\frac{1}{3}$ according to Copernicus, $60\frac{2}{5}$ according to Street, and $56\frac{1}{2}$ according to Tycho. But Tycho and all those who follow his tables of refractions, by making the refractions of the sun and moon (entirely contrary to the nature of light) be greater than those of the fixed stars—in fact greater by about four or five minutes—have increased the parallax of the moon by that many minutes, that is, by about a twelfth or fifteenth of the whole parallax. Let that error be corrected, and the distance will come to be roughly $60\frac{1}{2}$ terrestrial semidiameters, close to the value that

has been assigned by others. Let us assume a mean distance of 60 semidiameters in the syzygies; and also let us assume that a revolution of the moon with respect to the fixed stars is completed in 27 days, 7 hours, 43 minutes, as has been established by astronomers; and that the circumference of the earth is 123,249,600 Paris feet, according to the measurements made by the French. If now the moon is imagined to be deprived of all its motion and to be let fall so that it will descend to the earth with all that force urging it by which (by prop. 3, corol.) it is [normally] kept in its orbit, then in the space of one minute, it will by falling describe $15\frac{1}{12}$ Paris feet. This is determined by a calculation carried out either by using prop. 36 of book 1 or (which comes to the same thing) by using corol. 9 to prop. 4 of book 1. For the versed sine of the arc which the moon would describe in one minute of time by its mean motion at a distance of 60 semidiameters of the earth is roughly $15\frac{1}{12}$ Paris feet, or more exactly 15 feet, 1 inch, and $1\frac{1}{2}$ lines [or twelfths of an inch]. Accordingly, since in approaching the earth that force is increased as the inverse square of the distance, and so at the surface of the earth is 60×60 times greater than at the moon, it follows that a body falling with that force, in our regions, ought in the space of one minute to describe $60 \times 60 \times 15\frac{1}{12}$ Paris feet, and in the space of one second $15\frac{1}{12}$ feet, or more exactly 15 feet, 1 inch, and $1\frac{1}{2}$ lines. And heavy bodies do actually descend to the earth with this very force. For a pendulum beating seconds in the latitude of Paris is 3 Paris feet and $8\frac{1}{2}$ lines in length, as Huygens observed. And the height that a heavy body describes by falling in the time of one second is to half the length of this pendulum as the square of the ratio of the circumference of a circle to its diameter (as Huygens also showed), and so is 15 Paris feet, 1 inch, $1\frac{1}{2}$ lines. And therefore that force by which the moon is kept in its orbit, in descending from the moon's orbit to the surface of the earth, comes out equal to the force of gravity here on earth, and so (by rules 1 and 2) is that very force which we generally call gravity. For if gravity were different from this force, then bodies making for the earth by both forces acting together would descend twice as fast, and in the space of one second would by falling describe $30\frac{1}{2}$ Paris feet, entirely contrary to experience.

This calculation is founded on the hypothesis that the earth is at rest. For if the earth and the moon move around the sun and in the meanwhile also revolve around their common center of gravity, then, the law of gravity remaining the same, the distance of the centers of the moon and earth from

each other will be roughly $60\frac{1}{2}$ terrestrial semidiameters, as will be evident to anyone who computes it. And the computation can be undertaken by book 1, prop. 60.

The proof of the proposition can be treated more fully as follows. If several moons were to revolve around the earth, as happens in the system of Saturn or of Jupiter, their periodic times (by the argument of induction) would observe the law which Kepler discovered for the planets, and therefore their centripetal forces would be inversely as the squares of the distances from the center of the earth, by prop. 1 of this book 3. And if the lowest of them were small and nearly touched the tops of the highest mountains, its centripetal force, by which it would be kept in its orbit, would (by the preceding computation) be very nearly equal to the gravities of bodies on the tops of those mountains. And this centripetal force would cause this little moon, if it were deprived of all the motion with which it proceeds in its orbit, to descend to the earth—as a result of the absence of the centrifugal force with which it had remained in its orbit—and to do so with the same velocity with which heavy bodies fall on the tops of those mountains, because the forces with which they descend are equal. And if the force by which the lowest little moon descends were different from gravity and that little moon also were heavy toward the earth in the manner of bodies on the tops of mountains, this little moon would descend twice as fast by both forces acting together. Therefore, since both forces—namely, those of heavy bodies and those of the moons—are directed toward the center of the earth and are similar to each other and equal, they will (by rules 1 and 2) have the same cause. And therefore that force by which the moon is kept in its orbit is the very one that we generally call gravity. For if this were not so, the little moon at the top of a mountain must either be lacking in gravity or else fall twice as fast as heavy bodies generally do.

Scholium

The circumjovial planets [or satellites of Jupiter] gravitate toward Jupiter, the circumsaturnian planets [or satellites of Saturn] gravitate toward Saturn, and the circumsolar [or primary] planets gravitate toward the sun, and by the force of their gravity they are always drawn back from rectilinear motions and kept in curvilinear orbits.

Proposition 5
Theorem 5